

Relativistic Calculation of Pentaquark Widths

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Abstract

We calculate the widths of the various pentaquarks in a relativistic model in which the pentaquark is considered to be composed of a scalar diquark and a spin 1/2 triquark. We consider both positive and negative parity for the pentaquark. There is a single parameter in our model which we vary and which describes the size of the pentaquark. We obtain quite small widths for the decay $\Theta^+ \rightarrow N + K^+$ and for $\Theta_c^0 \rightarrow P + D^{*-}$ consistent with the experimental situation. For the sum of the decay widths for $\bar{\Xi}^{--} \rightarrow \Xi^- + \pi^+$ and $\bar{\Xi}^{--} \rightarrow \Sigma^- + K^-$ we find values of the order of 4 – 8 MeV for pentaquarks of the characteristic size considered in this work. (The experimental situation with respect to the observation of the $\bar{\Xi}^{--}$ is somewhat uncertain at this time.) We also provide results for the decays $N^+ \rightarrow N + \pi$ and $N_s^+ \rightarrow \Lambda^0 + K^+$. Our model of confinement plays an important role in our analysis and makes it possible to use Feynman diagrams to describe the decay of the pentaquark.

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I. INTRODUCTION

There has been a great deal of interest in the study of pentaquarks and a large number of experiments have been carried out [1-11]. The $\Theta^+(1540)$ which decays to a kaon and a nucleon has been seen in several experiments. It has been interpreted as a pentaquark with a $udud\bar{s}$ structure [12]. A pentaquark Θ_c^0 with the assumed structure $udud\bar{c}$ has also been observed recently. In the present work we will present calculations of the widths of several pentaquarks in a relativistic diquark-triquark model which includes a model of confinement that we have introduced previously in our study of meson and nucleon structure.

A number of reviews have appeared. In Refs.[13,14] the experimental evidence for the pentaquark is reviewed. Theoretical and experimental developments are reviewed in Refs.[15-18]. A number of theoretical papers have also appeared [12,19-24]. We are particularly interested in the work of Ref. [24] in which a diquark-triquark model is introduced to describe the pentaquark. We will make use of a variant of that model in the present work, since that model lends itself to the analysis of the pentaquark decay using Feynman diagrams. The diagram we consider is presented in Fig.1. There the pentaquark is represented by the heavy line with momentum P . The pentaquark is composed of a diquark of momentum $-k + P_N$ and a triquark of momentum $P + k - P_N$. The triquark emits a quark (u , d or s) which combines with the diquark to form a baryon of momentum P_N . The final-state meson of momentum $P - P_N$ is emitted along with the quark at the triquark vertex. (In the simplest model the triquark could be considered as composed of the final-state meson coupled to the exchanged quark, which ultimately forms part of the final-state baryon. [See Fig. 1.])

We have studied the structure of the nucleon in a quark-diquark model in Ref. [25]. In that work we considered both scalar and axialvector diquarks, however, in this work we will limit our considerations to a nucleon composed of a quark and a scalar diquark. The nucleon vertex is described in Ref. [25] for the case in which the diquark is placed on mass shell. (We remark that in Fig.1 only the quark of momentum k and the triquark of momentum $P + k - P_N$ will be off mass shell in our analysis.) The mass of the scalar diquark was taken to be 400 MeV in Ref.[25]. We use that value here when the diquark does not have a strange quark present. It is clear that we need to introduce a model of confinement for the pentaquark to carry out our program. Our (covariant) confinement model which we have

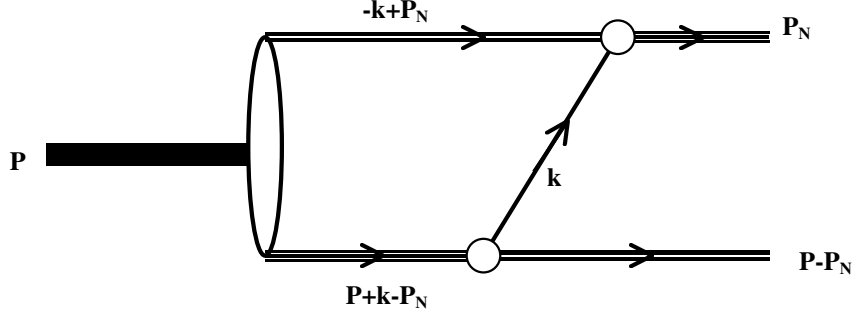


FIG. 1: In this figure the heavy line denotes the pentaquark and the line of momentum $-k + P_N$ denotes an on-mass-shell diquark. The line of momentum k represents the quark and $P + k - P_N$ is the momentum of the triquark. In the final state we have a baryon of momentum P_N and a meson of momentum $P - P_N$.

used extensively in other works [26-28] will be discussed in the next section.

The organization of our work is as follows. In Section II we will describe our model of confinement and in Section III we will discuss the widths of various pentaquarks of positive and negative parity. In Section IV we will provide the results of our numerical calculations and Section V contains some further discussion. The Appendix contains a discussion of the normalization of the various wave functions we have introduced.

II. A MODEL OF CONFINEMENT

For the moment, let us consider the case of the Θ^+ pentaquark with mass 1540 MeV. Our pentaquark is composed of a scalar diquark of mass 400 MeV and a triquark of mass 800 MeV. Since the mass of the pentaquark under consideration is 1540 MeV, the pentaquark would decay into its constituents in the absence of a model of confinement. Similarly, the nucleon of mass 939 MeV could decay into the scalar diquark of mass 400 MeV and a quark whose mass we take to be 350 MeV in this work.

In earlier work we have introduced a confining interaction which served to prevent the decay of mesons or nucleons into their constituents. Our covariant confinement model is

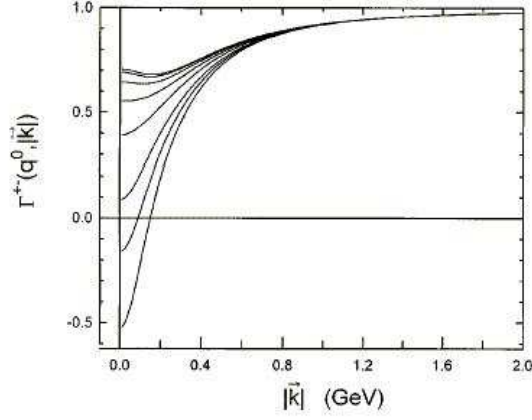


FIG. 2: Values of $\Gamma_S^{+-}(P^0, |\vec{k}|)$ defined in Ref. [29] are shown. Starting with the uppermost curve and moving downward, the values of P^0 are 0, 0.10, 0.20, 0.30, 0.40, 0.50, 0.55 and 0.60 GeV. For the last two of these curves $\Gamma_S^{+-}(P^0, k_{on}) = 0$. Here $k_{on}^2 = (P_0^2/2)^2 - m_q^2$, with $m_q = 0.260$ GeV.

described in a series of our papers [26-28]. In that model we solve a linear equation for a confining vertex function, Γ . This function has the following property. Consider the decay $A \rightarrow B + C$, in which the hadrons A and C are on mass shell. If we include the confining vertex we find the amplitude has a zero when particle B goes on mass shell, so that the amplitude for A to decay into two on-mass-shell particles (B and C) is zero. That feature may be seen in Fig.2 which is taken from Ref. [29].

For the decay $A \rightarrow B + C$ we may introduce a wave function that may be expressed in terms of the momentum of the off-shell particle B . If B is a scalar, we have

$$\Psi_B(k) = \frac{1}{k^2 - m_B^2} \Gamma_B(k). \quad (2.1)$$

Note that the ratio of $\Gamma_B(k)$ to $(k^2 - m_B^2)$ is an ordinary function which may often be well represented by a Gaussian function. (It is not necessary to include an $i\epsilon$ in the denominator of Eq. (2.1).)

In the case of the nucleon we may consider the decay into a quark and a diquark. In Ref. [25] we considered both scalar and axialvector diquarks, but for simplicity we will limit ourselves to only the scalar diquark. The relevant wave function in this case was given in Eq. (3.3) of Ref. [25]:

$$\Psi_S(P, k, s, t) = \tilde{\Psi}_{(1)}(P, k) \frac{2m_q \Lambda^{(+)}(\vec{k})}{\sqrt{2E(\vec{k}) (E(\vec{k}) + m_q)}} u_N(P, s) \chi_t \quad (2.2)$$

which we will simplify for the present work to read

$$\Psi_N(P_N, k, s, t) = \tilde{\Psi}_N(P_N, k) \Lambda^{(+)}(\vec{k}) u_N(P_N, s) \chi_t. \quad (2.3)$$

The function $\tilde{\Psi}_N(P, k)$ is represented in Fig. 5 of Ref. [25] by a dashed line. That function is well approximated by a Gaussian function which we will record at a latter point in our discussion. The factor of $\Lambda^{(+)}(\vec{k}) = (\not{k}_{on} + m_q)/2m_q$ arises from an approximation made in Ref.[25]. (Here $k_{on} = (E(\vec{k}), \vec{k})$ with $E(\vec{k}) = [\vec{k}^2 + m_q^2]^{1/2}$.) In Ref.[25] the quark propagator was written as

$$-iS(k) = \frac{m_q}{E_q(k)} \left[\frac{\Lambda^{(+)}(\vec{k})}{k^0 - E_q(\vec{k})} - \frac{\Lambda^{(-)}(-\vec{k})}{k^0 + E_q(\vec{k})} \right]. \quad (2.4)$$

The second term was neglected in our formalism when we studied the nucleon. (Thus we limited our analysis to positive-energy quark spinors.) Since we wish to make use of the nucleon wave function determined in Ref.[25], we will continue to include the projection $\Lambda^{(+)}(\vec{k})$ in our formalism. Here k is the quark momentum.

As stated earlier, the diquark of momentum $-k + P_N$ in Fig. 1 will be placed on mass shell, so that $k^0 = E_N(\vec{P}_N) - E_D(\vec{P}_N - \vec{k})$, where $E_D(\vec{P}_N - \vec{k}) = [(\vec{P}_N - \vec{k})^2 + m_D^2]^{1/2}$ and $E_N(\vec{P}_N) = [\vec{P}_N^2 + m_N^2]^{1/2}$. That approximation is achieved by writing

$$\frac{1}{(P_N - k)^2 - m_D^2 + i\epsilon} \longrightarrow -2\pi i \delta^{(+)}[(P_N - k)^2 - m_D^2], \quad (2.5)$$

as described in detail in Ref.[30]. Note that

$$\delta^{(+)}[(P_N - k)^2 - m_D^2] = \frac{1}{2E_D(\vec{P}_N - \vec{k})} \delta[P_N^0 - k^0 - E_D(\vec{P}_N - \vec{k})]. \quad (2.6)$$

The on-mass-shell specification used here arises when performing an integral in the complex k^0 plane [30].

III. CALCULATION OF THE WIDTHS OF NEGATIVE AND POSITIVE PARITY PENTAQUARKS

We consider the diagram shown in Fig. 1. Recall that the heavy line of momentum P denotes the pentaquark. The line carrying momentum $-k + P_N$ is the on-mass-shell scalar diquark and the line with momentum $P + k - P_N$ is the triquark. The momentum k is that of an up, down or strange quark, $P - P_N$ is the momentum of the final-state baryon. The

pentaquark, final-state nucleon and final-state meson are all on mass shell. In our analysis the diquark is also on mass shell, so that only the triquark and quark propagate off mass shell, as noted earlier. (As stated in the last section, the on-shell characterization of the diquark arises when we complete the k^0 integral in the complex k^0 plane.)

We now make use of the formula [31]

$$d\Gamma = |\mathfrak{M}|^2 \frac{d^3 k_1}{(2\pi)^3} \frac{m_N}{E_N(\vec{k}_1)} \frac{d^3 k_2}{(2\pi)^3} \frac{1}{2E_K(\vec{k}_2)} (2\pi)^4 \delta^4(P - k_1 + k_2), \quad (3.1)$$

where \vec{k}_1 and \vec{k}_2 are the momenta of the outgoing particles. We may put $\vec{k}_1 = \vec{P}_N$ and $\vec{k}_2 = \vec{P} - \vec{P}_N = -\vec{P}_N$ for a pentaquark at rest. (It is convenient to take \vec{P}_N along the z axis when calculating the width.)

In writing our expression for Γ we will represent the product of the quark propagator and nucleon vertex function by the nucleon wave function of Eq. (2.3). In a similar fashion we will represent the product of the triquark propagator and the pentaquark-triquark vertex function by a triquark wave function. (In the vertex, the diquark is on mass shell.) We then have to specify the vertex function of the triquark which describes the decay into the quark of momentum k and the final-state meson. That scalar part of the vertex is usefully written as

$$\Gamma_T(k) = \Psi_T(k)(k^2 - m_q^2). \quad (3.2)$$

Thus, the wave functions $\Psi_N(k)$, $\Psi_T(k)$ and $\Psi_\Theta(P_N - k)$ will appear in our expression for Γ_Θ when we consider the decay of the Θ^+ . Note that for the decay $\Theta^+ \rightarrow N + K^+$, we have

$$\vec{P}_N^2 = \left(\frac{m_\Theta^2 - m_N^2 + m_K^2}{2m_\Theta} \right)^2 - m_K^2 \quad (3.3)$$

which yields $\vec{P}_N^2 = 0.0722 \text{ GeV}^2$, or $|\vec{P}_N| = 0.269 \text{ GeV}$.

We find that the width is given by

$$\begin{aligned} \Gamma_\Theta = & \frac{1}{2} \int \frac{d\vec{k}}{(2\pi)^3} \frac{1}{2E_D(\vec{k} - \vec{P}_N)} \int \frac{d\vec{k}'}{(2\pi)^3} \frac{1}{2E_D(\vec{k}' - \vec{P}_N)} \frac{1}{N_\Theta N_T N_N} \frac{1}{(2m_N)(2m_\Theta)} \\ & \times \Psi_N(\vec{k}) \Psi_N(\vec{k}') \Psi_\Theta(\vec{P}_N - \vec{k}) \Psi_\Theta(\vec{P}_N - \vec{k}') (k^2 - m_q^2) \Psi_T(\vec{k}) (k'^2 - m_q^2) \Psi_T(\vec{k}') \\ & \times \text{Tr} \left[(\not{k} - \not{P}_N + \not{P} + m_T) (\not{k}_{on} + m_q) (\not{P}_N + m_N) \gamma^0 \right. \\ & \left. (\not{k}'_{on} + m_q) (\not{k}' - \not{P}_N + \not{P} + m_T) \gamma^0 (\not{P} + m_\Theta) \right] \rho, \end{aligned} \quad (3.4)$$

where ρ is the phase space factor. We find for the decay of the Θ^+ ,

$$\rho = \frac{1}{2\pi} \frac{m_N}{m_\Theta} |\vec{P}_N| \quad (3.5)$$

$$= 0.0261 \text{ GeV}, \quad (3.6)$$

since $|\vec{P}_N| \simeq 0.269 \text{ GeV}$.

The factor $\not{P}_N + m_N$ and $\not{P} + m_\Theta$ in the trace arise from the relations

$$\frac{\not{P}_N + m_N}{2m_N} = \sum_{s_N} u_N(\vec{P}_N, s_N) \bar{u}_N(\vec{P}_N, s_N) \quad (3.7)$$

and

$$\frac{\not{P} + m_\Theta}{2m_\Theta} = \sum_s u_\Theta(\vec{P}, s) \bar{u}_\Theta(\vec{P}, s). \quad (3.8)$$

The factors $(1/N_\Theta)^{1/2}$, $(1/N_T)^{1/2}$ and $(1/N_N)^{1/2}$ serve to normalize the wave functions. (See the Appendix.) We determine that $N_N = 0.316$, $N_T = 0.0982$, and calculate N_Θ for each choice of the pentaquark wave function. We write, with $k = |\vec{k}|$,

$$\Psi_N(\vec{k}) = \frac{1}{\sqrt{N_N}} \left(y_0 + \frac{A}{w_0 \sqrt{\frac{\pi}{2}}} e^{-\frac{2(k - k_c)^2}{w_0^2}} \right) \quad (3.9)$$

and

$$\Psi_T(\vec{k}) = \frac{1}{\sqrt{N_T}} \left(y_0 + \frac{A}{w_0 \sqrt{\frac{\pi}{2}}} e^{-\frac{2(k - k_c)^2}{w_0^2}} \right). \quad (3.10)$$

We have determined y_0 , A , k_c and w_0 from a fit to the wave function given in Fig. 5 of Ref.[25]. We find $y_0 = -3.66$, $k_c = -0.013 \text{ GeV}$, $w_0 = 0.660 \text{ GeV}$ and $A = 27.73 \text{ GeV}$. For the pentaquark we write

$$\Psi_\Theta(\vec{k} - \vec{P}_N) = \frac{1}{\sqrt{N_\Theta}} \frac{A}{w_\Theta \sqrt{\frac{\pi}{2}}} e^{-\frac{2(\vec{k} - \vec{P}_N)^2}{w_\Theta^2}}, \quad (3.11)$$

where w_Θ is a variable in our analysis. (Note that N_Θ depends upon the choice of w_Θ .) Here $\vec{k} - \vec{P}_N$ is the relative momentum of the diquark and triquark when $\vec{P} = 0$.

In the case of a positive-parity pentaquark we assume that the pentaquark decays to a positive-parity diquark and a positive-parity triquark. As before, the triquark and diquark have zero relative angular momentum. In this case we need to insert factors of $i\gamma_5$ at the triquark-meson vertex where the quark of momentum k is emitted. In addition, the calculation of the normalization factor N_T is modified. (See the Appendix.) In Eq. (3.4), the trace becomes

$$\begin{aligned} \text{Trace} = \text{Tr} [& (k - \not{P}_N + \not{P} + m_T) i\gamma_5 (\not{k}_{on} + m_q) (\not{P}_N + m_N) \gamma^0 \\ & (\not{k}'_{on} + m_q) i\gamma_5 (\not{k}' - \not{P}_N + \not{P} + m_T) \gamma^0 (\not{P} + m_\Theta)] . \end{aligned} \quad (3.12)$$

We may define $\tilde{k} = \gamma^0 k \gamma^0$, etc. Thus

$$\begin{aligned} \text{Trace} = \text{Tr} [& (k - \not{P}_N + \not{P} + m_T) \gamma_5 (\not{k}_{on} + m_q) (\not{P}_N + m_N) (\tilde{k}'_{on} + m_q) \\ & \gamma_5 (\tilde{k}' - \tilde{\not{P}}_N + \tilde{\not{P}} + m_T) (\not{P} + m_\Theta)] \end{aligned} \quad (3.13)$$

$$\begin{aligned} = \text{Tr} [& (k - \not{P}_N + \not{P} + m_T) (-\not{k}_{on} + m_q) (-\not{P}_N + m_N) (-\tilde{k}'_{on} + m_q) \\ & (\tilde{k}' - \tilde{\not{P}}_N + \tilde{\not{P}} + m_T) (\not{P} + m_\Theta)] . \end{aligned} \quad (3.14)$$

In this case we find $N_N = 0.316$ and $N_T = 0.0201$, where only the second value has changed relative to the values given above.

IV. NUMERICAL RESULTS

In this section we present the results of our numerical calculations. In Fig. 3 we exhibit the width for the decay $\Theta^+ \rightarrow N + K^+$ in the case that the pentaquark has positive parity. The result for a negative-parity pentaquark is shown in Fig. 4. The parameters used are given in the caption of Fig. 3. The small value for the diquark mass used here corresponds to the analysis made in Ref. [29]. In the case of the negative parity pentaquark (Fig. 4) very small widths may be obtained for w in the range $0.75 \text{ GeV} < w < 0.80 \text{ GeV}$. For the case of positive parity, quite small widths are found in the range $0.70 \text{ GeV} < w < 0.90 \text{ GeV}$. [See Fig. 3.]

Results for the decay $\Theta_c^0 \rightarrow P + D^{*-}$ are given in Figs. 5 and 6. Again, we find very small widths for the negative-parity pentaquark in the range $0.75 \text{ GeV} < w < 0.80 \text{ GeV}$ with widths of about $3 \rightarrow 4 \text{ MeV}$ for the positive-parity Θ_c^0 in the corresponding range. (The parameters used are given in the figure captions.)

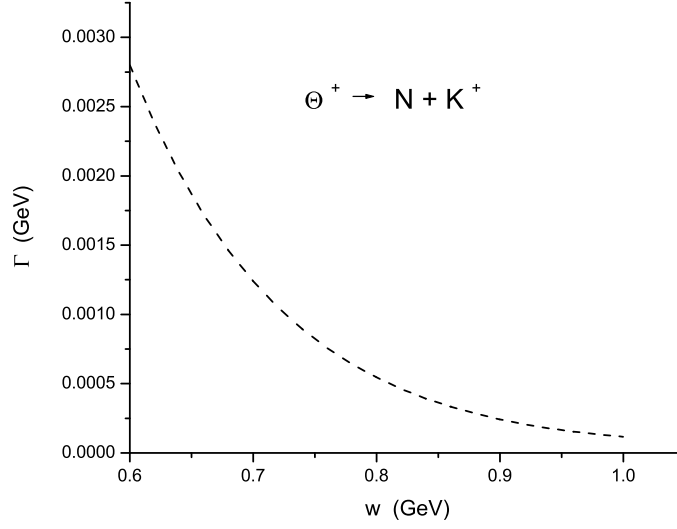


FIG. 3: The width for the decay $\Theta^+ \rightarrow N + K^+$ for a positive-parity pentaquark as calculated in our model is shown. Here $m_\Theta = 1.540$ GeV, $m_T = 0.800$ GeV, $m_D = 0.400$ GeV, $m_N = 0.939$ GeV and $m_K = 0.495$ GeV.

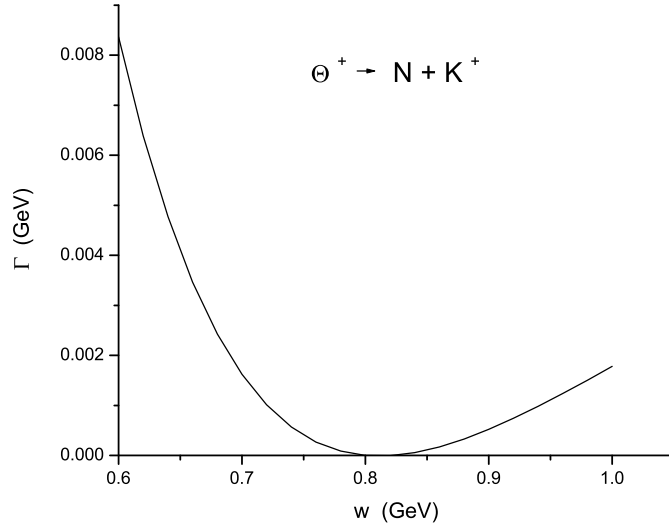


FIG. 4: The width of the negative parity pentaquark is shown as a function of the parameter w . (The quite small values in the vicinity of the minimum have rather large uncertainties because of the limitation of the number of points used in our five-dimensional integral which determines the width.) The parameters used are given in the caption to Fig. 3.

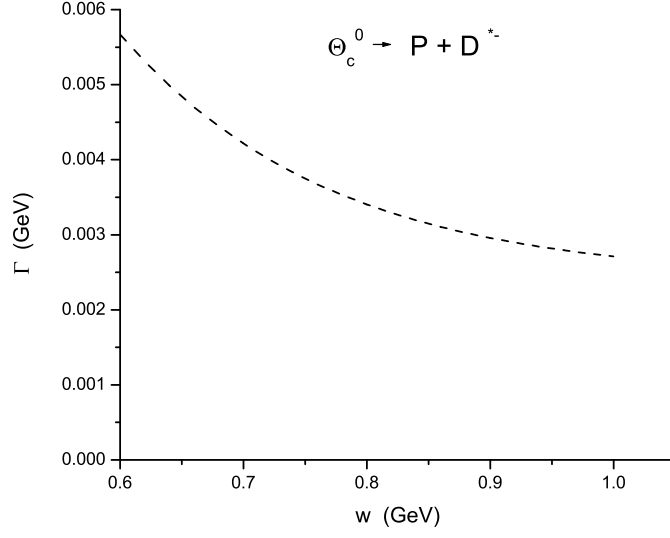


FIG. 5: The calculated width of the positive-parity charmed pentaquark is shown for the decay $\Theta_c^0 \rightarrow P + D^{*-}$. Here. $m_{\Theta_c^0}=3.099$ GeV, $m_P=0.939$ GeV, $m_D = 0.400$ GeV, $m_T = 2.00$ GeV, $m_q = 0.350$ GeV and $m_{D^{*-}} = 2.007$ GeV.

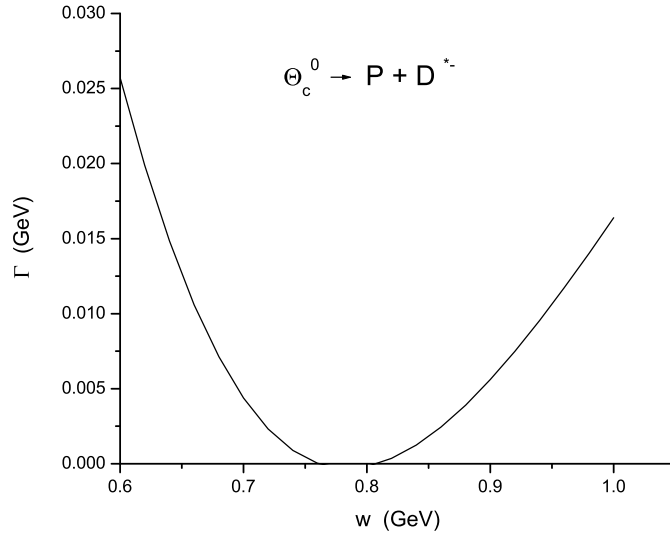


FIG. 6: The calculated width of the negative-parity charmed pentaquark is shown. The calculation is made for the parameters given in the caption of Fig. 5.

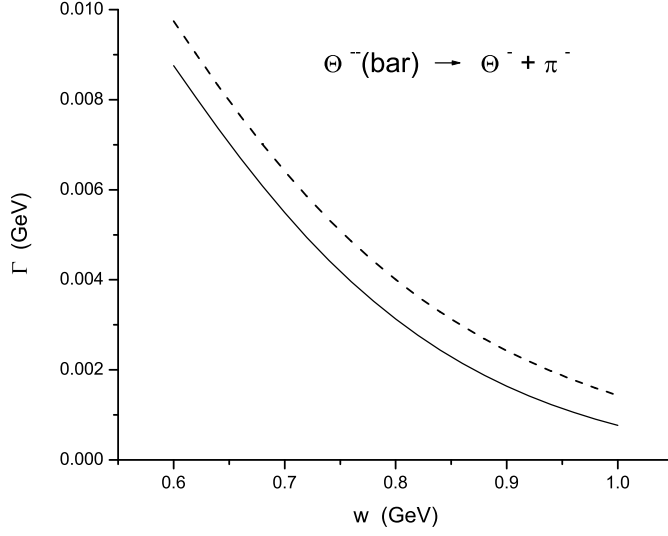


FIG. 7: The calculated width for the decay $\bar{\Xi}^{--} \rightarrow \Xi^- + \pi^-$ is shown for the case that the $\bar{\Xi}^{--}$ has positive parity (dashed line) and for negative parity (solid line). The parameters used are $m_{\bar{\Xi}}=1.862$ GeV, $m_T=0.800$ MeV, $m_D = 0.700$ GeV, $m_s = 0.450$ GeV, $m_{\Xi} = 1.321$ GeV and $m_{\pi} = 0.139$ GeV.

As stated earlier, the observation of the $\bar{\Xi}^{--}$ pentaquark is a matter of some controversy. Its observation was reported in Ref. [3] with a width less than 18 MeV, however, that state was not seen in the work reported in Refs. [5] and [11]. Our results calculated for the decay $\bar{\Xi}^{--} \rightarrow \Xi^- + \pi^-$ are shown in Fig. 7, while our results for the decay $\bar{\Xi}^{--} \rightarrow \Sigma^- + K^-$ are given in Fig. 8. In the case of a negative parity $\bar{\Xi}^{--}$ the summed results for the two decay channels are given in Fig.9 (negative-parity pentaquark) and Fig.10 (positive-parity pentaquark). The calculated widths are of the order of 4-8 MeV in the region $0.75 \text{ GeV} < w < 0.80 \text{ GeV}$. The $\bar{\Xi}^{--}$ is not seen in some experiments that involved a search for this pentaquark [5, 11].

In Fig. 11 we present our results for the decay $N^+ \rightarrow N + \pi^+$. Here we use the mass suggested in Ref. [12] for the N^+ . In this case very small widths are calculated. Much larger widths for the N^+ decay to $\Lambda^0 + K^+$ are found and the calculated values are exhibited in Fig. 12. Since the N^+ and N_s^+ have the quantum numbers of the nucleon, they are expected to mix with neighboring states and be hard to distinguish from other nucleon resonances. The mixing of such states is discussed in Ref. [32].

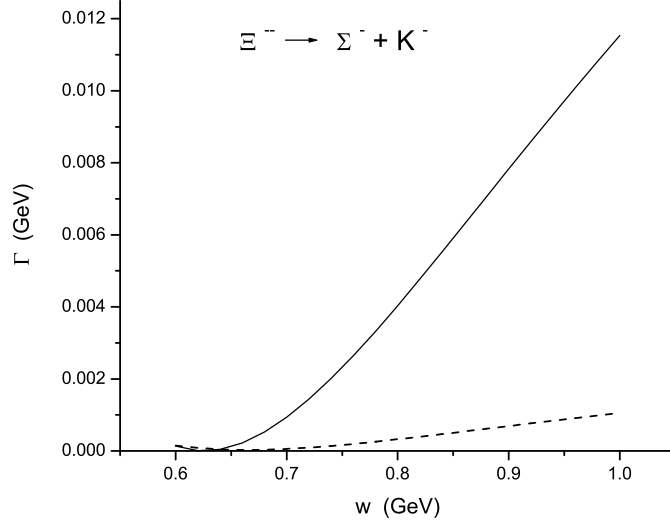


FIG. 8: The calculated width for the decay $\bar{\Xi}^{--} \rightarrow \Sigma^- + K^-$ is shown for the case that the $\bar{\Xi}^{--}$ has positive parity (dashed line) and for negative parity (solid line). The parameters used for $m_{\bar{\Xi}}$, m_T , m_D , and m_q are given in the caption to Fig. 7. Here $m_{\Sigma^-} = 1.197$ GeV and $m_K = 0.495$ GeV.

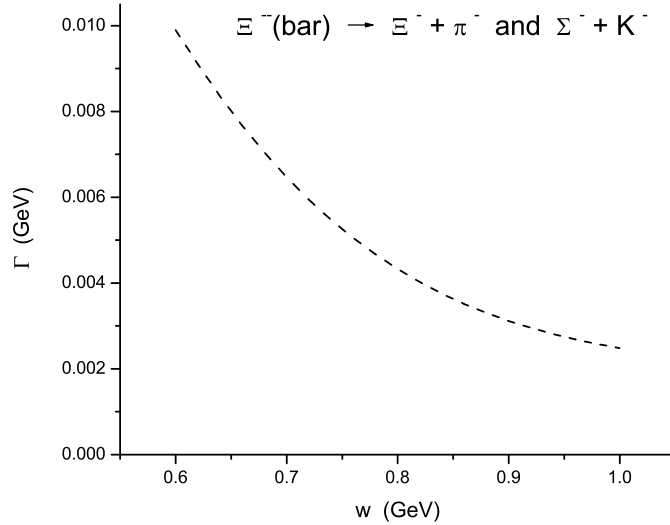


FIG. 9: The calculated widths for the decays $\bar{\Xi}^{--} \rightarrow \Xi^- + \pi^-$ and $\Sigma^- + K^-$ are summed and shown in the figure. Here the $\bar{\Xi}^{--}$ is assumed to have positive parity. The parameters used are given in the captions of Figs. 7 and 8.

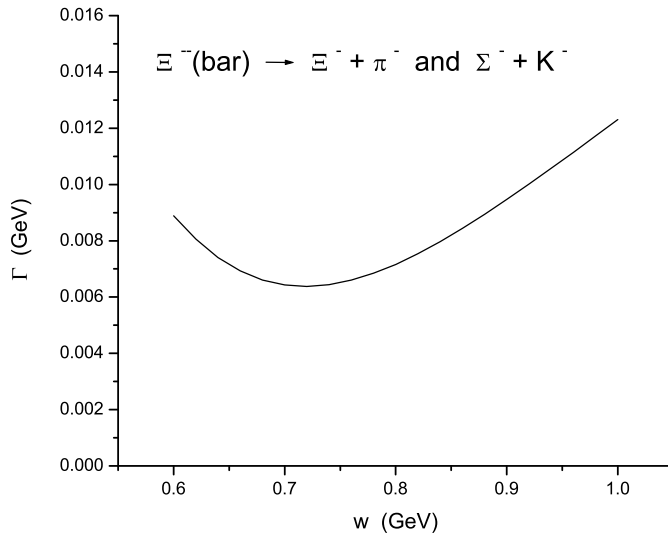


FIG. 10: Same caption as Fig. 9 except that the Ξ^{--} is taken to have positive parity.

V. DISCUSSION

It is of interest to note that the same computer code can yield different values for the pentaquark widths depending upon the mass values inserted for the pentaquark, final-state baryon and meson, and for the diquark and triquark. We do recognize that our model is limited since we have not taken into account the full symmetry of the wave function. That is, we do not consider the identity of the quarks in the diquark and the triquark. It is possible that a more complex calculation made in the future may overcome this limitation of the model. On the other hand, our results are quite suggestive, since very small widths are obtained for the Θ^+ and Θ_c^0 in accordance with the experimental situation. It is clear that further experimental studies and theoretical analysis is desirable.

APPENDIX A: NORMALIZATION PARAMETER FOR THE NEGATIVE-PARITY AND POSITIVE-PARITY PENTAQUARK STATES

The normalization of the final-state baryon wave function is calculated by requiring that that the baryon contain a single quark in addition to the scalar diquark. In that calculation the diquark appears on mass shell. We find in the case of a nucleon in the final state, the

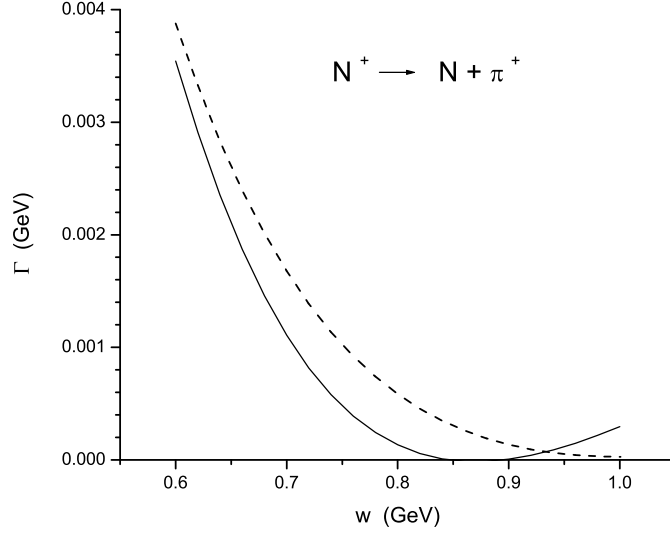


FIG. 11: The values obtained for the decays $N^+ \rightarrow N + \pi^+$ are shown for a positive-parity N^+ (dashed line) and for negative parity (solid line). The parameters used were $m_N = 0.939$ GeV, $m_\pi = 0.139$ GeV, $m_{N^+} = 1.440$ GeV, $m_T = 0.800$ GeV and $m_d = 0.350$ GeV.

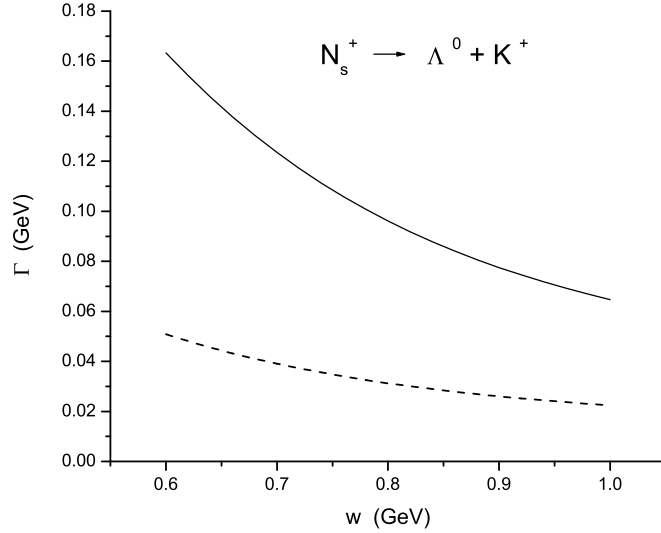


FIG. 12: The calculated width for the decays $N_s^+ \rightarrow \Lambda^0 + K^+$ are shown for a positive parity N_s^+ (dashed line) and for negative parity (solid line). The parameters used were $m_{\Lambda^0} = 1.116$ GeV, $m_{N_s} = 1.700$ GeV, $m_D = 0.400$ GeV, $m_s = 0.450$ GeV, $m_K = 0.495$ GeV and $m_T = 1.00$ GeV.

normalization parameter is

$$N_N = \frac{1}{2\pi^2} \int_0^{k_{max}} \frac{k^2 dk}{E_D(\vec{k})} (k k^0) (k^0 + m_q) |\Psi_N(\vec{k})|^2, \quad (\text{A1})$$

where $E_D(\vec{k}) = (\vec{k}^2 + m_D^2)^{1/2}$, with m_D being the mass of the diquark and $k^0 = m_N - E_D(\vec{k})$. Here we take $k_{max} = 0.7$ GeV. In Eq. (A1)

$$\Psi_N(\vec{k}) = \left(y_0 + \frac{A}{w_0 \sqrt{\frac{\pi}{2}}} \exp^{-\frac{2(k-k_c)^2}{w_0^2}} \right). \quad (\text{A2})$$

[See Eq. (3.9).] Values of y_0, w_0, k_c and A are given after Eq. (3.10).

The calculation of the normalization factor for the pentaquark yields a similar result. For example

$$N_\Theta = \frac{1}{2\pi^2} \int_0^{k_{max}} \frac{k^2 dk}{E_D(\vec{k})} k^0 k (k^0 + m_T) |\Psi_\Theta(\vec{k})|^2, \quad (\text{A3})$$

with $k^0 = m_\Theta - E_D(\vec{k})$. Here $E_D(\vec{k}) = (\vec{k}^2 + m_D^2)^{1/2}$, with m_D being the mass of the diquark. For the pentaquark, we use the wave function

$$\Psi_\Theta(\vec{k}) = \frac{A}{w \sqrt{\frac{\pi}{2}}} \exp^{-\frac{2(\vec{k}-\vec{P}_N)^2}{w^2}} \quad (\text{A4})$$

in Eq. (A3). [See Eq. (3.11).] Here w is a variable of our model. Results for various values of w are given in the figures for different pentaquarks.

The triquark normalization factor is calculated by requiring that the triquark contain a single quark in addition to the final-state meson which is placed on mass shell in the calculation. We find

$$N_T = \frac{1}{4\pi^2} \int_0^{k_{max}} k^2 dk (k_0^2 + \vec{k}^2 + 2k_0 m_q + m_q^2) |\tilde{\Psi}_T(\vec{k})|^2, \quad (\text{A5})$$

where $k_0 = m_T - E_K(\vec{k})$. Here $E_K = (\vec{k}^2 + m_K^2)^{1/2}$, where m_K is the mass of the final-state meson, and m_q is the mass of the exchanged quark. [See Fig. 1.] Note that, in this case,

$$\Psi_T(\vec{k}) = y_0 + \frac{A}{w_0 \sqrt{\frac{\pi}{2}}} \exp^{-\frac{2(k-k_c)^2}{w_0^2}}, \quad (\text{A6})$$

with the parameters y_0, w_0, k_c and A given after Eq. (3.10).

If we consider positive parity pentaquark states, we do not have to change the baryon normalization parameter which was given in Eq. (A1).

The normalization parameter of the pentaquark wave function is still of the form given in Eq. (A2), while the triquark normalization factor is now

$$N_T = \frac{1}{4\pi^2} \int_0^{k_{max}} k^2 dk (k_0^2 + \vec{k}^2 - 2k_0 m_q + m_q^2) | \tilde{\Psi}_T(\vec{k}) |^2 . \quad (\text{A7})$$

Comparison to Eq. (A5) shows that the sign of the term $2k_0 m_q$ in Eq. (A7) has changed relative to the sign in Eq. (A5).

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